

R09

Code No: 09A1BS01

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, November/December - 2013

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours

Max. Marks: 75

Answer any five questions
All questions carry equal marks

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1.a) Test the convergence of the following series:

i) $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \sqrt{\frac{3}{8}} + \dots + \sqrt{\frac{n}{2(n+1)}} + \dots$

ii) $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n \quad (x > 0)$

b) Define conditional convergence and absolute convergence.

Test the following series for convergence and absolute convergence:

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots \quad [15]$$

2.a) Examine the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ for extreme values.

b) Write the statement of Rolle's mean value theorem and its Geometrical interpretation. Give two examples of functions and the corresponding intervals where the Rolle's Theorem fails. [15]

3.a) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3 + y^3 = 3axy$.b) Trace the curve $x = a(\theta + \sin \theta), y = a(1 + \cos \theta)$. [15]4.a) Evaluate $\iint_R (x + y) dy dx$, R is the region bounded by $x = 0, x = 2, y = x, y = x + 2$.b) Evaluate $\iiint xyz(x^2 + y^2 + z^2) dx dy dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing the variables to spherical polar coordinates. [15]5.a) Solve $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0$.b) Show that the family of parabolas $y^2 = 2cx + c^2$ is "self-orthogonal". [15]6.a) Solve $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x (1 + 2 \tan x)$.b) Apply the method of variation of parameters to solve $\frac{d^2 y}{dx^2} + y = \tan x$. [15]

- 7.a) Using Convolution Theorem, evaluate $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$.
- b) Using Laplace transformation solve the following differential equation:
 $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0)=1$, $x'(0)=0$ [15]
- 8.a) If $\vec{A} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate the line integral $\oint \vec{A} \cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the curve C: $x=t, y=t^2, z=t^3$.
- b) Use Stoke's Theorem to evaluate $\int_c \vec{v} \cdot d\vec{r}$, where $\vec{v} = y^2\vec{i} + xy\vec{j} + xz\vec{k}$, and c is the bounding curve of the hemisphere $x^2 + y^2 + z^2 = 9, z > 0$, oriented in the positive direction. [15]

